Review

Factoring:

$(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

$(a+b)(a-b) = a^2 - b^2$

$27x^5 + 9x^4 = 9x^4(3x+1)$

$9x^2 - 25 = (3x+5)(3x-5)$

$\sqrt{9x^2} = \sqrt{9}\cdot\cancel{\sqrt{x^2}} = 3x$

Solve quadratic equations: $A \cdot B = 0$

$\overset{a}{1x^2} + \overset{b}{7x} + \overset{c}{12} = 0$

$AC = 12$, $B = 7$ $\longrightarrow$ $\boxed{3 \,\&\, 4}$

prod. sum

$\underbrace{(x+3)}_{A}\underbrace{(x+4)}_{B} = 0$

$x + 3 = 0 \longrightarrow \boxed{x = -3}$ or $x + 4 = 0 \longrightarrow \boxed{x = -4}$

$-3 \quad -3 \qquad\qquad -4 \quad -4$

$x = \dfrac{-b \pm \sqrt{\Delta}}{2a}$, $\Delta = b^2 - 4ac$

$\begin{cases} \Delta > 0 : 2 \text{ sol'ns} \\ \Delta = 0 : 1 \text{ sol'n} \end{cases}$

$\Delta < 0$: no sol'ns

$\sqrt{x^2} = \pm\sqrt{\dfrac{16}{25}}$ or $x^2 - \dfrac{16}{25} = 0$

$x = \pm\dfrac{\sqrt{16}}{\sqrt{25}} \longrightarrow x = \pm\dfrac{4}{5}$

Class 3: Section 1.6 - Other Types of Equations

→ exponents > 2, neg. exponents, root equations

If $a \neq 0$, then $a^{-n} = \frac{1}{a^n}$, for an integer n.

$$a^{-n} = \left(\frac{a}{1}\right)^{-n} = \left(\frac{1}{a}\right)^{n} = \frac{1}{a^n}$$

Examples:

$$k^{-5} = \frac{1}{k^5}$$

$$\frac{1}{\underset{a}{(-2)}^{\overset{n}{-4}}} = (-2)^4 = 16$$

$$\left(\frac{3}{4}\right)^{-2} = \left(\frac{4}{3}\right)^{2} = \frac{4^2}{3^2} = \frac{16}{9}$$

a any and n integer, then $a^{\frac{1}{n}} = \sqrt[n]{a}$.

Examples:

$$\underbrace{64}_{a}^{\frac{1}{2} \leftarrow n} = \sqrt[2]{64} = 8$$

$$64^{\frac{1}{3}} = \sqrt[3]{64} = 4 \rightarrow 64 \wedge (1/3)$$

$$\boxed{a^{-n} = \frac{1}{a^n}}$$

$$x^{-1} = -\frac{1}{4} \rightarrow x = -4$$

$$\boxed{x^{-1} = \frac{1}{x}}$$

$$\frac{1}{x^1} = -\frac{1}{4}$$

$$\frac{1}{x^{-1}} = -2 \rightarrow \boxed{\frac{1}{x^{-1}} = x} \quad \boxed{x = -2}$$

- Higher degree equations

Ex1: $\boxed{-\frac{1}{4}x\left(x^2 - 49\right)^2 = 0}$

$$-\frac{1}{4}x^5 + \frac{49}{2}x^3 - \frac{2401}{4}x = 0$$

$$-\frac{1}{4}x\left(x^4 - 98x^2 + 2401\right) = 0$$

$$\underbrace{-\frac{1}{4}x}_{A}\underbrace{\left(x^2 - 49\right)^2}_{B} = 0$$

$$\frac{\cancel{-\frac{1}{4}}x}{\cancel{-\frac{1}{4}}} = \frac{0}{-\frac{1}{4}}$$

$$\boxed{x = 0}$$

or

$$\sqrt{\left(x^2 - 49\right)^{\cancel{2}}} = \sqrt{0}$$

$$x^2 - 49 = 0$$
$$\quad +49 \quad +49$$

$$\sqrt{x^2} = \sqrt{49} \rightarrow \boxed{x = \pm 7}$$

Ex 2: Find all solutions: $\underbrace{x^3 - x^2}\ \underbrace{-\ 11x + 11} = 0.$

$x^2(x-1) - 11(x-1) = 0$

$\underbrace{(x-1)}_{A}\underbrace{(x^2-11)}_{B} = 0$

Solution set:

$\{-\sqrt{11}, 1, \sqrt{11}\}$

$x - 1 = 0$ or $x^2 - 11 = 0$

$+1 \quad +1 \qquad\qquad +11 \quad +11$

$\boxed{x = 1}$

$\sqrt{x^2} = \sqrt{11} \longrightarrow \boxed{x = \pm\sqrt{11}}$

Ex 3: Find all solutions for $6c^3 + 18c^2 - 168c = 0.$

$\underbrace{6c}_{A}\underbrace{(c^2 + 3c - 28)}_{B} = 0$

product   add

$AC = -28$ & $B = +3$

$\underline{\underline{+7}}$ & $\underline{\underline{-4}}$

$\frac{6c}{6} = \frac{0}{6}$ or $\overset{A}{1c^2} + \overset{B}{3c} - \overset{C}{28} = 0$

$\boxed{c = 0}$

$\underline{c^2 + 7c}\ \underline{- 4c - 28} = 0$

$\underbrace{c(c+7)}\ \underbrace{-4(c+7)} = 0$

$c + 7 = 0$

$-7 \quad -7$

$\boxed{c = -7}$

$(c+7)(c-4)$

$\frac{c(\cancel{c+7})}{\cancel{c+7}}$ & $\frac{-4(\cancel{c+7})}{\cancel{c+7}}$

$\underbrace{(c+7)}_{A}\underbrace{(c-4)}_{B} = 0$

$c - 4 = 0 \longrightarrow \boxed{c = 4}$

$+4 \quad +4$

Ex 4: Find solutions for $\underbrace{x^2(x^2-81)}\ \underbrace{-16(x^2-81)} = 0.$

$\underbrace{(x^2-81)}_{A}\underbrace{(x^2-16)}_{B} = 0 \Rightarrow x^2 - 81 = 0$ or $x^2 - 16 = 0$

$+81 \quad +81 \qquad\qquad +16 \quad +16$

$x^2 = 81 \Rightarrow \boxed{x = \pm 9} \qquad x^2 = 16 \Rightarrow \boxed{x = \pm 4}$

$(-9)^2\left((-9)^2-81\right)-16\left((-9)^2-81\right)=0$

$81(81-81)-16(81-81)=0$ $\Rightarrow$ -9 is a solution.

$81\cdot 0 \quad -16\cdot 0 \quad = 0$

$0-0=0$ ✓

→ Root equations

Ex 1: $\sqrt{3x-6}=3$

$\left(\cancel{\sqrt{3x-6}}\right)^{\cancel{2}}=(3)^2 \Rightarrow 3x-6=9 \Rightarrow \frac{3x}{3}=\frac{15}{3} \Rightarrow \boxed{x=5}$

$\qquad\qquad\qquad\qquad\quad +6 \quad\; +6$

$\sqrt{3\cdot 5-6}=3 \Rightarrow \sqrt{15-6}=3 \Rightarrow \sqrt{9}=3$ ✓

Ex 2: $\sqrt{5-4x}-x=0.$

$\qquad\qquad\qquad\quad +x \quad +x$

Step 1: the whole sqrt on right/left.

Step 2: square it.

Step 3: solve for x.

Step 4: check ans.

$\left(\sqrt{5-4x}\right)^2=(x)^2$

$\cancel{5-4x=x^2}$

$-5 \quad +4x \quad +4x-5$

$0=x^2+4x-5$

$\left.\begin{array}{l} ac=-5 \\ b=4 \end{array}\right\} \; 5 \;\&\; -1$

$0=\underbrace{(x+5)}_{A}\underbrace{(x-1)}_{B}$ or $\begin{cases} x+5=0 \Rightarrow \boxed{x=-5} ✗ \\ x-1=0 \Rightarrow \boxed{x=1} ✓ \end{cases}$

$\sqrt{5-4x}-x=0 \rightarrow x=1: \sqrt{5-4\cdot 1}-1=0 \Rightarrow \sqrt{5-4}-1=0$ ✓

$\sqrt{5-4\cdot(-5)}-(-5)=0 \Rightarrow \sqrt{5+20}+5=0 \Rightarrow \sqrt{25}+5=0$ ✗

Exercise: Find solutions for $x-5=\sqrt{17x+25}$

$(a-b)^2=a^2-2ab+b^2$

1. Placement
2. Square
3. solve
4. check

$\boxed{(x-5)^2}=\left(\sqrt{17x+25}\right)^{\cancel{2}}$

$(x-5)^2=(x-5)(x-5)$
$=x^2-5x-5x+25$
$=x^2-10x+25$

$\Rightarrow x^2-10x+25=17x+\cancel{25}$
$\quad -17x-25 \quad \cancel{-17x}-25$

$\Rightarrow x^2-27x=0$

$\Rightarrow \underbrace{x}_{A}\underbrace{(x-27)}_{B}=0$

OR $\begin{cases} x=0 & \text{X} \\ x-27=0 \Rightarrow x=27 & ✓ \end{cases}$

$x-5=\sqrt{17x+25}$

$x=0$: $0-5=\sqrt{17\cdot 0+25}$
$-5=\sqrt{25}$ FALSE

$x=27$: $27-5=\sqrt{17\cdot 27+25}$
$22=\sqrt{459+25}$
$22=\sqrt{484}$ ✓

"Disguised Quadratic Equations"

Ex 1: $x^{4}-18x^{2}+32=0.$

$ax^{2}+bx^{1}+c=0$

$u=x^2$: $(x^2)^2-18(x^2)+32=0$

$u^2-18u+32=0 \Rightarrow (u-16)(u-2)=0 \Rightarrow \boxed{\begin{matrix} u=16 \\ u=2 \end{matrix}}$

$x^2=16 \Rightarrow \boxed{x=\pm 4}$

OR

$x^2=2 \Rightarrow \boxed{x=\pm\sqrt{2}}$

Step 1: identify $\underline{u}$.

Step 2: solve for $\underline{u}$.

Step 3: solve for $\underline{x}$.

Ex2: $3x^{\frac{2}{3}} - 7x^{\frac{1}{3}} + 2 = 0$

$3\left(x^{\frac{1}{3}}\right)^2 - 7\left(x^{\frac{1}{3}}\right) + 2 = 0$

Step 1: identify u. $\boxed{u = x^{\frac{1}{3}}}$ $\longrightarrow$ $3u^2 - 7u + 2 = 0.$

$ac = 6,\ b = -7 \rightarrow -1 \ \& -6$

step2: solve for u.

$\Rightarrow \underbrace{3u^2 - u}\ \underbrace{-6u + 2} = 0$

$\Rightarrow \boxed{u}(3u-1)\boxed{-2}(3u-1) = 0$ OR

$\Rightarrow \underbrace{(3u-1)}_{a}\underbrace{(u-2)}_{b} = 0.$

$3u - 1 = 0 \Rightarrow \frac{3u}{3} = \frac{1}{3} \Rightarrow \boxed{u = \frac{1}{3}}$
$\quad +1 \quad +1$

$u - 2 = 0 \Rightarrow \boxed{u = 2}$
$\quad +2 \quad +2$

Step 3: solve for x. $\boxed{x^{\frac{1}{3}} = u}$ $\quad u = 2$, $u = \frac{1}{3}$

$\left(x^{\frac{1}{3}}\right)^3 = (2)^3 \Rightarrow \boxed{x = 8}$ OR $\left(x^{\frac{1}{3}}\right)^3 = \left(\frac{1}{3}\right)^3 \Rightarrow \boxed{x = \frac{1}{27}}$

$\left(x^{\frac{1}{3}}\right)^3 = \left(2^1\right)^3 \rightarrow x^1 = 2^3$

Ex 3: $3\left(\frac{1}{x-1}\right)^2 - \frac{8}{x-1} - 3 = 0$

$3\left(\frac{1}{x-1}\right)^2 - 8\left(\frac{1}{x-1}\right) - 3 = 0$

$\boxed{u = \frac{1}{x-1}} \Rightarrow \overset{A}{3}u^2 \overset{B}{-8}u \overset{C}{-3} = 0$

$AC = -9$ (prod), $B = -8$ (sum) $\rightarrow 1 \ \& -9$

$\underbrace{3u^2 - 9u}\ \underbrace{+ u - 3} = 0$

$3u(u-3) + 1(u-3) = 0$

$(u-3)(3u+1) = 0 \rightarrow \boxed{u = 3, -\frac{1}{3}}$

$3(x-1) = \frac{1}{\cancel{(x-1)}}\cancel{(x-1)} \rightarrow 3x - 3 = 1$

$3x = 4 \Rightarrow \boxed{x = \frac{4}{3}}$

$-\frac{1}{\cancel{3}} \cdot \cancel{3}(x-1) = \frac{1}{\cancel{x-1}} \cdot 3\cancel{(x-1)}$

$-(x-1) = 3 \Rightarrow -x + 1 = 3$
$\qquad\qquad\qquad -1 \quad -1$

$\frac{\cancel{-}x}{\cancel{-1}} = \frac{2}{-1} \Rightarrow \boxed{x = -2}$

Ex 4: $3x^{-2} - 5x^{-1} - 2 = 0.$

1^{st}: identify $\underline{u}$: $\boxed{u = x^{-1}}$ $\quad 3(x^{-1})^2 - 5(x^{-1}) - 2 = 0$

2^{nd}: solve for $\underline{u}$:

$$\underset{A}{3u^2} - \underset{B}{5u} - \underset{C}{2} = 0$$

$$\left.\begin{array}{ll} AC = -6 & \text{prod} \\ B = -5 & \text{sum} \end{array}\right\} \; 1 \;\&\; -6$$

$$\underbrace{3u^2 + u}\underbrace{-6u - 2} = 0$$

$$u(3u+1) - 2(3u+1) = 0$$

$$\underbrace{(3u+1)}_{a}\underbrace{(u-2)}_{b} = 0$$

$$u = -\frac{1}{3}, \quad u = 2$$

3^{rd}: solve for $\underline{x}$:

$$\left(-\frac{1}{3}\right)^{-1} = \left(x^{-1}\right)^{-1} \rightarrow \left(-\frac{3}{1}\right)^{1} = x^{1} \text{ or } \boxed{x = -3}$$

$$(2)^{-1} = \left(x^{-1}\right)^{-1} \rightarrow \left(\frac{1}{2}\right)^{1} = x^{1} \text{ or } \boxed{x = \frac{1}{2}}$$